Integration — free sample worksheet — answers

Mathematics 9709 · Topic 1.8 · 20 marks · Mark scheme and how not to lose marks

Question 1

EASY [3 marks]

Find $\int \left(\frac{2}{x^3} + \sqrt{x}\right) dx$.

Answer

Rewrite as powers: $\int \left(2x^{-3} + x^{1/2}\right) dx = \frac{2x^{-2}}{-2} + \frac{x^{3/2}}{3/2} + c = -\frac{1}{x^2} + \frac{2}{3}x^{3/2} + c$

Mark scheme

M1 rewriting as $2x^{-3}$ and $x^{1/2}$
A1 one term correct
A1 fully correct, including $+c$

> **How not to lose marks**
> Rewrite fractions and roots as powers *before* integrating — and show that line: it is the method mark.

Question 2

EASY [2 marks]

Find $\int (3x^2 - 4x + 5)\, dx$.

Answer

$x^3 - 2x^2 + 5x + c$

Mark scheme

M1 increase a power by 1 and divide by the new power (at least two terms)
A1 $x^3 - 2x^2 + 5x + c$, including $+c$

> **How not to lose marks**
> Write "$+c$" the moment you integrate. Leaving it off an indefinite integral costs the accuracy mark even when everything else is perfect.

Question 3

MEDIUM [3 marks]

A curve has gradient $\frac{dy}{dx} = 3x^2 - 2$ and passes through the point $(1, 4)$. Find the equation of the curve.

Answer

$y = x^3 - 2x + c$. At $(1, 4)$: $4 = 1 - 2 + c$, so $c = 5$. The curve is $y = x^3 - 2x + 5$.

Mark scheme

M1 integrate, with $+c$
M1 substitute $(1, 4)$ to find c
A1 $y = x^3 - 2x + 5$

> **How not to lose marks**
> Without $+c$ there is nothing to find, and every later mark is lost. Finish by writing the full equation "$y = \ldots$" — not just $c = 5$.

Question 4

MEDIUM [3 marks]

Find $\displaystyle\int \frac{6}{(2x+1)^3}\,dx$.

Answer

$$\int 6(2x+1)^{-3}\,dx = \frac{6(2x+1)^{-2}}{(-2)(2)} + c = -\frac{3}{2(2x+1)^2} + c$$

Mark scheme

M1 integrate to $k(2x+1)^{-2}$
M1 divide by -2 and by 2 (the derivative of $2x+1$)
A1 $-\dfrac{3}{2(2x+1)^2} + c$

> **How not to lose marks**
> For $(ax+b)^n$, divide by the new power *and* by a. Check by differentiating your answer — it takes ten seconds and catches the missing factor of 2.

Question 5

HARD [5 marks]

The curve $y = x^2 - 4x + 3$ crosses the x-axis at $x = 1$ and $x = 3$. Find the total area enclosed between the curve, the x-axis and the lines $x = 0$ and $x = 3$.

Answer

$$\int (x^2 - 4x + 3)\,dx = \frac{x^3}{3} - 2x^2 + 3x.$$

From 0 to 1: $\left(\frac{1}{3} - 2 + 3\right) - 0 = \frac{4}{3}$ (above the axis).

From 1 to 3: $(9 - 18 + 9) - \frac{4}{3} = -\frac{4}{3}$ (below the axis, so the area is $\frac{4}{3}$).

Total area $= \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$.

Mark scheme

M1 correct integration
M1 split the integral at $x = 1$, where the curve crosses the axis
A1 $\frac{4}{3}$ for 0 to 1
A1 $-\frac{4}{3}$ for 1 to 3, treated as an area of $\frac{4}{3}$
A1 total $\frac{8}{3}$

> **How not to lose marks**
> Integrating straight from 0 to 3 gives 0 — the area below the axis cancels the area above. Sketch the curve first; wherever it crosses the axis, split the integral and add the areas as positives.

Question 6

HARD [4 marks]

The region bounded by the curve $y = \dfrac{3}{2x+1}$, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through $360°$ about the x-axis. Find the exact volume of the solid formed.

Answer

$$V = \pi \int_0^1 \frac{9}{(2x+1)^2}\,dx = \pi \left[-\frac{9}{2(2x+1)}\right]_0^1 = \pi\left(-\frac{9}{6} + \frac{9}{2}\right) = 3\pi$$

Mark scheme

M1 $V = \pi \int y^2\,dx$ with y squared correctly
M1 integrate to $k(2x+1)^{-1}$
A1 $-\dfrac{9}{2(2x+1)}$
A1 3π (exact)

> **How not to lose marks**
> "Exact" means leave π in: 3π, not 9.42. And square y *before* integrating — $\pi \int y\,dx$ is the most common wrong start.